Ain Shams Engineering Journal 13 (2022) 101666

Contents lists available at ScienceDirect

Ain Shams Engineering Journal

journal homepage: www.sciencedirect.com

Ten non-polynomial cubic splines for some classes of Fredholm integral equations

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ARTICLE INFO

Article history:
Received 28 September 2021
Revised 30 November 2021
Accepted 15 December 2021

Keywords:
Non-polynomial cubic spline method
Fredholm integral equations
Fuzzy Fredholm integral equation

ABSTRACT

In this paper, a new general form for the non-polynomial cubic spline function is introduced. This form enables us to generate about ten different formulas of the proposed spline function. Although some of these functions have been used before to solve some types of differential and integral equations, the others are considered new. These ten functions are used to approximate the solution of linear, nonlinear, and fuzzy Fredholm integral equations of second kind. Moreover, the convergence analysis of the proposed method is established which indicates that its order of convergence is four. Also, some examples are solved and compared with the previous methods which showed that the present method is more accurate.

1. Introduction

Consider the linear Fredholm integral equation (LFIE) of second kind [1–15] and the nonlinear Fredholm integral equation (NLFIE) [13–23] take the following forms, respectively:

$$y(x) = f(x) + \int_a^b \psi(x,t)y(t)dt, \tag{1.1}$$

$$y(x) = f(x) + \int_a^b \psi(x,t,y(t))dt, \qquad a \le x \le b, \tag{1.2}$$

where $f(x), \psi(x,t)$ and $\psi(x,t,y(t))$ are given continuous functions defined on $[a,b]$ and $y(x)$ is the unknown function. The fuzzy Fredholm integral equation (FFIE) [25–28] is represented by equation (1.1) when $\psi(x,t)$ is a bivariate function over the square $a \le x,t \le b$ and $f(x)$ is a given fuzzy function.

Many numerical techniques are used to investigate the solutions of LFIE, NLFIE and FFIE such as quarter–sweep Guass–Seidel method (QSGSM) [1], functional approximation method (FAM) [2]), quasi–interpolation method (QIM) [3], quarter–sweep iteration method (QSIM) [6]), triangular functions method (TFM) [8], collocation method (CM) [13], Newton–Kantorovich–quadrature method (NKQM) [17], discrete adomian decomposition method (DADM) [22], discrete homotopy analysis method (DHAM) [23], block–pulse functions (BPFs) [26] ... etc. For more details, Muthuvalu and Sulaiman [1] studied the solution of LFIE using QSGSM which gave better result than half and full sweep Guass–Seidel methods. Long and Nelakanti [2] used FAM to solve it while Müller and Varnhorn [3] solved it using QIM. Also, Bellour et al. [4] discussed its solution using natural cubic spline approximation and cubic spline quasi–interpolation method. Panda et al. [5] introduced a modified approach depends on the quadrature rule to approximate the integral in equation (1.1) and the Lagrange interpolation to approximate this function (MA–QRLI). In addition, Mohamad and Sulaiman [6] and Almasieh and Roodaki [8] gave a piecewise collocation solution for FIE by QSIM [6] and TFM [8]. Tohidi [9] used Taylor matrix method to solve 2D LFIE. Lemita et al. [10–11] applied a new process to approach equation (1.1) defined on large interval while Guebbai [12] used regularization and Fourier series to solve it. Ebrahimi and Rashidinia [13], Zhong [14] and Rashidinia et al. [15] investigated the solution of both LFIE and NLFIE using CM [13], integral mean value method (IMVM) [14] and non-polynomial spline method (NPSM) [15]. A novel numerical method depends on the integral mean–value theorem used by Li and Huang [16] to solve equation (1.2). Nadjafi and Heidari [17] solved NLFIE by NKQM. Furthermore, Aziz and Islam [19] determined the solution of it using Haar–wavelets method while

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Peer review under responsibility of Ain Shams University.

Production and hosting by Elsevier

https://doi.org/10.1016/j.asej.2021.101666
2090-4479/